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ERROR ANALYSIS OF A VARIABLE-POWER DIVIDER

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY
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ERROR ANALYSIS OF A VARIABLE-POWER DIVIDER

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Group 61

TECHNICAL NOTE 1975-32

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LEXINGTON MASSACHUSETTS

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ABSTRACT

Errors in the division of power by a variable-power divider using phase shifters and a pair of 3-dB couplers are determined as a function of an incorrect setting of the phase shifters. The insertion phase, of the phase shifters, is assumed to have a Gaussian distribution about the desired value and the resultant probability distribution of the power division is calculated. Curves are given for the probability that P_A falls within a given error. These curves are computed from the probability density function of P_A which is also shown graphically.

Error Analysis of a Variable-Power Divider

A variable-power dividing four-port junction consisting of a magic "tee", a 3-dB-short-slot coupler, and two latching-ferrite phase-shifters, as indicated schematically in Fig. 1, has output signals given by the following equations:

$$E_A = \sin\left[\frac{\phi_1-\phi_2}{2} - \frac{\pi}{4}\right] e^{j\left(\frac{\phi_1+\phi_2}{2} + \frac{\pi}{4}\right)} \qquad (1)$$

$$E_B = \cos\left[\frac{\phi_1-\phi_2}{2} - \frac{\pi}{4}\right] e^{j\left(\frac{\phi_1+\phi_2}{2} + \frac{\pi}{4}\right)} \qquad (2)$$

where ϕ_1 and ϕ_2 are the insertion phase shift of the latching-phase shifters. These equations assume the magic tee and 3-dB coupler are perfect lossless devices, an assumption that is reasonable in view of the comparative error in establishing ϕ_1 and ϕ_2.

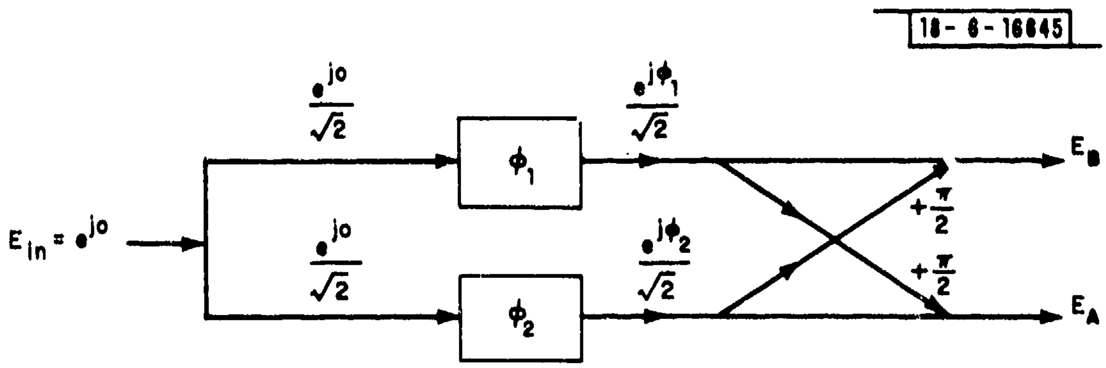

Fig. 1. Variable-Power Divider.

Equations (1) and (2) can be put in a somewhat simpler form if we assume ϕ_2 varies from $\frac{\pi}{2}$ to π and ϕ_1 varies from 0 to $\frac{\pi}{2}$; that is ϕ_1 has a minimum insertion phase of $\frac{\pi}{2}$ radians. In addition we let $\xi = \phi_1 - \phi_2 - \pi/2$, and consider only the power delivered to the output ports since the output signals are always in phase (because we also assume the same insertion loss for each phase shifter) and our interest centers on the device as a power divider. Hence

$$P_A = \sin^2 \frac{\xi}{2} \tag{3}$$

and

$$P_B = \cos^2 \frac{\xi}{2} \tag{4}$$

The length, τ, of a pulse applied to the ferrite phase shifter determines its insertion phase. The relationship between insertion phase and pulse width is non-linear; it varies as a function of temperature and aging, and each device is also somewhat different from all other "identical" devices. The problem addressed in this note concerns determining the error in P_A and P_B when a polynomial representation such as;

$$\tau = \sum_{n=0}^{N} a_n \phi^n \tag{5}$$

is used to set the phase shifters, in any variable power divider (VPD), to obtain a desired division of power.

Toward this end let us first determine a method of obtaining (5), or its inverse

$$\phi = \sum_{n=0}^{N} a_n \tau^n, \tag{5a}$$

and what probability density function (pdf) we should assign to the dependent variable. Although it is customary to determine τ for a given power division by using (5) and (3), τ is ultimately applied to the control equipment so as to obtain the desired power division. Hence it is more practical to consider τ as the independent variable and use (5a) and (3) or (4) to evaluate the error in P_A and P_B.

In particular, let us determine the insertion phase, ϕ_{mn} of a large number of phase shifters say $M = 40$ or more, (m = phase shifter number, n = drive pulse duration) for a finite set of error free applied control pulses ($\tau_1, \tau_2 \ldots \tau_N$) and several temperatures and frequencies in the operating range. Assuming ϕ_{mn}, for a particular value of $n = i$, to have a Gaussian distribution about the mean value, $\overline{\phi}_i$, ($\overline{\phi}_i = (1/M) \sum_{m=1}^{M} \phi_{mi}$), the standard deviation σ_i and $\overline{\phi}_i$ are computed. (In a most exact analysis one would attempt to determine the actual frequency function, or pdf, but the assumed Gaussian and perhaps a uniform distribution should establish an interesting bound as will be discussed later.) The values of $\overline{\phi}_i$ and $\overline{\tau}_i$ are then fit to a pth degree polynomial by a suitable regression analysis and the a_n of (5a) are determined. It may also be useful to determine the b_m, in

$$\sigma = \sum_{m=0}^{M} b_m \tau^m \tag{6}$$

in the same manner. Hence (5a) and (6), together with assumed Gaussian distribution, define the statistical performance of the phase shifters as a group.

For a Gaussian distribution of $\phi(\tau_i)$, for fixed τ_i, the linear combination

$$\xi = \phi(\tau_\ell) - \phi(\tau_m) \tag{7}$$

is also a Gaussian distribution with mean $\overline{\xi} = \overline{\phi}_\ell - \overline{\phi}_m$ and variance $\sigma^2 = \sigma_\ell^{\,2} + \sigma_m^{\,2}$.

It remains to find the pdf of $p_A(u)$, and then the probability that P_A lies between $P_A + \Delta P_A$ and $P_A - \Delta P_A$. Toward this end we recognize that the probability density function of ξ is given by

$$p_\xi(x) = \frac{e^{-\frac{1}{2}\left(\frac{x-\overline{\xi}}{\sigma}\right)^2}}{\sqrt{2\pi}\;\sigma} \tag{8}$$

The relationship between P_A, or P_B and ξ is obtained from (3) and (4) and rewritten here as

$$P_A = \frac{1}{2} - \frac{1}{2}\cos\xi = 1-P_B \tag{9a}$$

$$P_B = \frac{1}{2} + \frac{1}{2}\cos\xi \tag{9b}$$

The probability $x_1 < P_A < x_2$ is given by

$$P[x_1 < P_A < x_2] = \int_{x_1}^{x_2} p_A(u)\,du \tag{10}$$

where x_1 and x_2 are assumed values of P_A.

Sketching (8) and (9b) (see Fig. 2a and 2b),

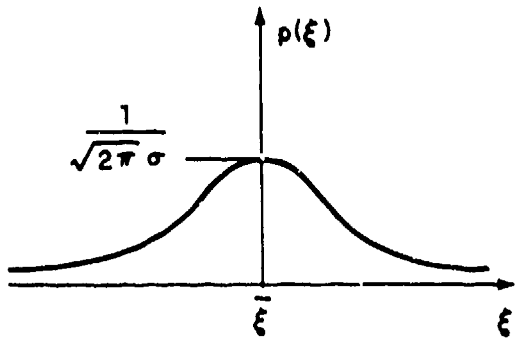

Fig. 2a. Probability density function of ξ.

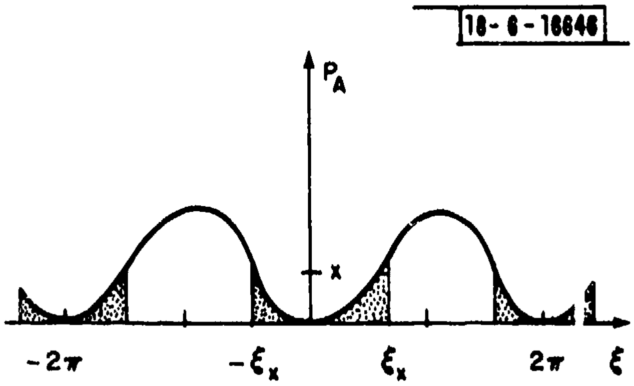

Fig. 2b. Distribution of P_A.

it is apparent that P_A is a periodic function of ξ and obtains a value less than x for all ξ between $-\xi_x + 2n\pi$ and $\xi_x + 2n\pi$ where n is any integer between $-\infty$ and ∞. Hence from Figs. 2a and 2b we can write

$$P[P_a < x] = \sum_{n=-\infty}^{\infty} \int_{-\xi_x+2n\pi}^{\xi_x+2n\pi} p_\xi(u)\,du \tag{13}$$

where from (9a)

$$\xi_x = \cos^{-1}[1-2x] \ . \tag{14}$$

If we limit $\overline{\xi}$ to the range $0 \leq \overline{\xi} \leq \pi$ all interesting values of P_A will be obtained; i.e., $0 \leq P_A \leq 1$. We should also recognize that σ in (8) is ≈ 0.1. Consequently for $0 < \overline{\xi} < \pi/2$ $(0 < x < 1/2)$, only the $n = 0$ term in (13) will be significant. For $\pi/2 < \overline{\xi} < \pi$ $(1/2 < x < 1)$, the $n = 1$ term must also be taken into account. Specifically

$$P[P_A < x] = \int_{-\xi_x}^{\xi_x} p_\xi(u)\,du \tag{13a}$$

for $0 < x < \frac{1}{2}$ and

$$P[P_A < x] = \int_0^{\xi_x} p_\xi(u)\,du + \int_{2\pi-\xi_x}^{2\pi} p_\xi(u)\,du \qquad (13b)$$

for $\frac{1}{2} < x < 1$. Note that (13b) neglects the contribution for $-\xi_x < \xi < 0$ and $2\pi < \xi < 2\pi + \xi_x$ because they are negligible.

Recalling the definition

$$p_A(x) = \lim_{\Delta P_A \to 0} \frac{P[x < P_A < x + \Delta P_A]}{\Delta P_A} = \frac{d}{dx}\Big(P[P_A < x]\Big) \qquad (15)$$

and using (13a), we have

$$p_A(x) = p_\xi(\xi_x)\,\frac{d\xi_x}{dx} - p_\xi(-\xi_x)\,\frac{d(-\xi_x)}{dx} \quad , \; 0 \le x \le \frac{1}{2} \qquad (16a)$$

Using (13b) we have

$$p_A(x) = p_\xi(\xi_x)\,\frac{d\xi_x}{dx} - p_\xi(2\pi-\xi_x)\,\frac{d(-\xi_x)}{dx} \quad , \; \frac{1}{2} \le x \le 1 \qquad (16b)$$

From (14), we have

$$\frac{d\xi_x}{dx} = \frac{1}{\sqrt{x(1-x)}} \qquad (17)$$

Substituting (17) and (8) into (16a) and (16b) we consider the combination resulting from (16a) and (16b) to be

$$p_A(x) = \frac{1}{\sigma\sqrt{2\pi}} \left[\frac{e^{-\frac{1}{2}\left(\frac{\xi_x-\overline{\xi}}{\sigma}\right)^2} + e^{-\frac{1}{2}\left(\frac{\xi_x+\overline{\xi}}{\sigma}\right)^2} + e^{-\frac{1}{2}\left(\frac{-\xi_x-\overline{\xi}+2\pi}{\sigma}\right)^2}}{\sqrt{x(1-x)}} \right]. \quad (18)$$

It is clear that only the first term on the right hand side will contribute for $0 < \overline{\xi} < \pi$. The second term contributes when ξ_x and $\overline{\xi}$ approach 0 and the third term contributes only when $\overline{\xi}$ and ξ_x approach π. As we would expect $p_A(x)$ has its maximum value for $\xi_x \approx \overline{\xi}$. The probability density function $p_A(x)$ is sketched in Fig. 3 with $P_A' = \sin^2\overline{\xi}$ as a parameter.

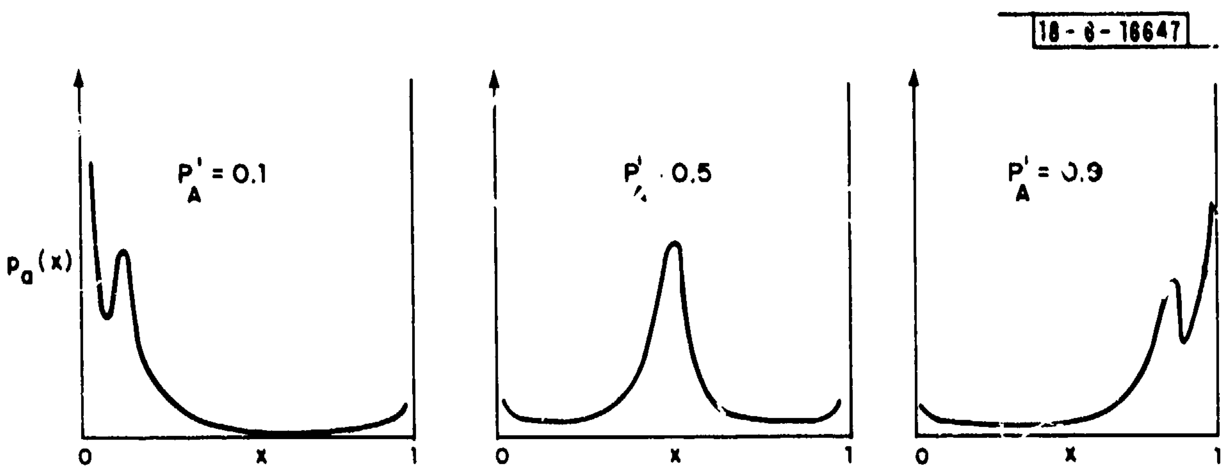

Fig. 3. Probability density function.

Note that the pdf has singularities at x = 0 and 1. However, it is an integrable singularity and the area under curves shown, over the range $0 < x < 1$, is finite and equals 1 as it should.

For example let us evaluate the area over a region $(0,\varepsilon)$ about the $x = 0$ singularity, i.e.,

$$P[P_A < \varepsilon] = \int_0^{\varepsilon} p_A(x)\,dx \tag{19}$$

where $\varepsilon \to 0$. Using (18) we have

$$P[P_A < \varepsilon] \approx \frac{1}{\sigma\sqrt{2\pi}} \int_0^{\varepsilon} \left(\frac{e^{-\frac{1}{2}\left(\frac{2\sqrt{x}-\bar{\xi}}{\sigma}\right)^2} + e^{-\frac{1}{2}\left(\frac{2\sqrt{x}+\bar{\xi}}{\sigma}\right)^2}}{\sqrt{x}} \right) dx \tag{20}$$

where we have set $\xi_x = 2\sqrt{x}$ (see (14)) and $\sqrt{x(1-x)} = \sqrt{x}$. Rearranging (20)

$$P[P_A < \varepsilon] \approx \frac{e^{-\frac{1}{2}\left(\frac{\bar{\xi}}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}} \int_0^{\varepsilon} \frac{e^{-\frac{2(x+\sqrt{x}\bar{\xi})}{\sigma^2}} + e^{-\frac{2(x-\bar{\xi}\sqrt{x})}{\sigma^2}}}{\sqrt{x}}\,dx. \tag{21}$$

For $x \to 0$ the numerator of the exponent can be approximated as indicated in (22)

$$P[P_A < \varepsilon] \approx \frac{e^{-\frac{1}{2}\left(\frac{\bar{\xi}}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}} \int_0^{\varepsilon} \frac{2 - \frac{4x}{\sigma^2} + \frac{2x^2}{\sigma^4} + \frac{2x\bar{\xi}^2}{\sigma^2}}{\sqrt{x}}\,dx \tag{22}$$

$$\approx \frac{e^{-\frac{1}{2}\left(\frac{\bar{\xi}}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}} \left[4\varepsilon^{1/2} - \frac{4(2-\bar{\xi})\varepsilon^{3/2}}{3\sigma^2} + \frac{4\varepsilon^{5/2}}{5\sigma^2} \right] \tag{23}$$

Now if $\varepsilon << \sigma^2$ (recall $0 < \bar{\xi} < \pi$), the second and third terms (23) can be neglected and

$$P[P_A < \varepsilon] \approx \frac{e^{-\frac{1}{2}\left(\frac{\overline{\xi}}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}} \; 4\sqrt{\varepsilon}. \tag{24}$$

Notice that (24) gives a finite probability indicating that integration over the singularity at $x = 0$ does indeed give a finite value. With $\sigma = 0.1$ and $\overline{\xi} = 0.2$

$$P[P_A < \varepsilon] \approx 2\sqrt{\varepsilon} \tag{25}$$

where $\varepsilon < .001$. Conversely $P[P_A > \varepsilon] = 1 - 1.5\sqrt{\varepsilon}$. This result will be checked in the next section.

Cumulative Distribution of P_A

Using the foregoing expression (18), and using numerical integration,

$$P[P_A < x] = \int_0^x p_A(u)\,du \tag{26}$$

was calculated for $\sigma = 0.05$, 0.1 and 0.2 and $P_A' = 0.01$, 0.5, 0.99. P_A' is the desired value of P_A; it is not necessarily the mean, or average, value of P_A. The results are shown in Figs. 4, 5 and 6. The $P[P_A' - x < P_A < P_A' + x]$ is plotted versus magnitude of $\Delta x = (P_A' - x)$ in Figs. 7, 8 and 9.

Notice that for $P_A' = .01$ (Fig. 4), which corresponds to $\overline{\xi} = .2$, $P[P_A < .001] \approx .07$ which agrees reasonably well with the approximate relationship given by Eq. (25). It is also important to note the asymmetrical character of the curves with respect to $x' = P_A' = .01$ the desired value of P_A. For $\sigma = .2$

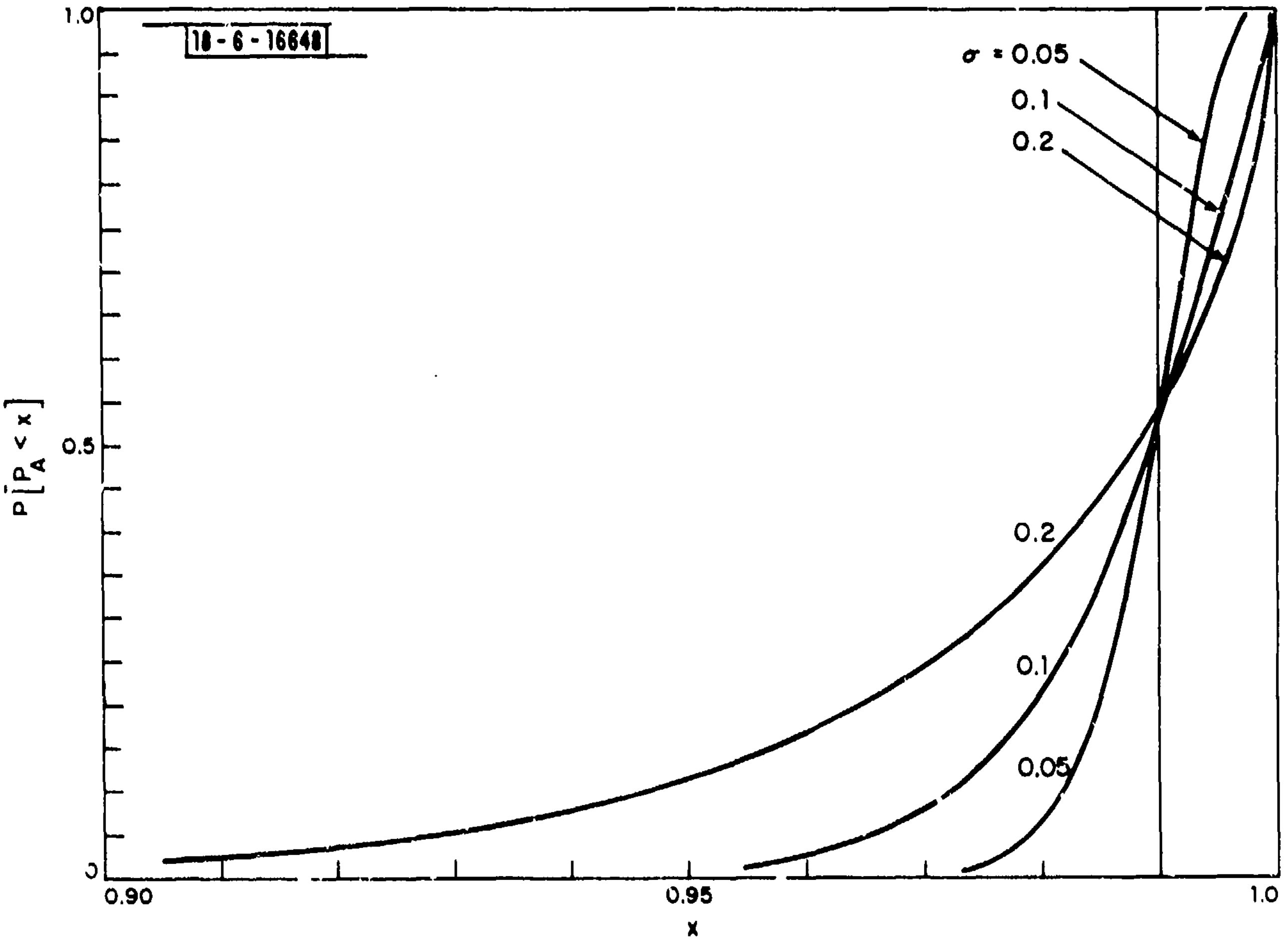

Fig. 4. Cumulative distribution of P_A for $P'_A = 0.01$.

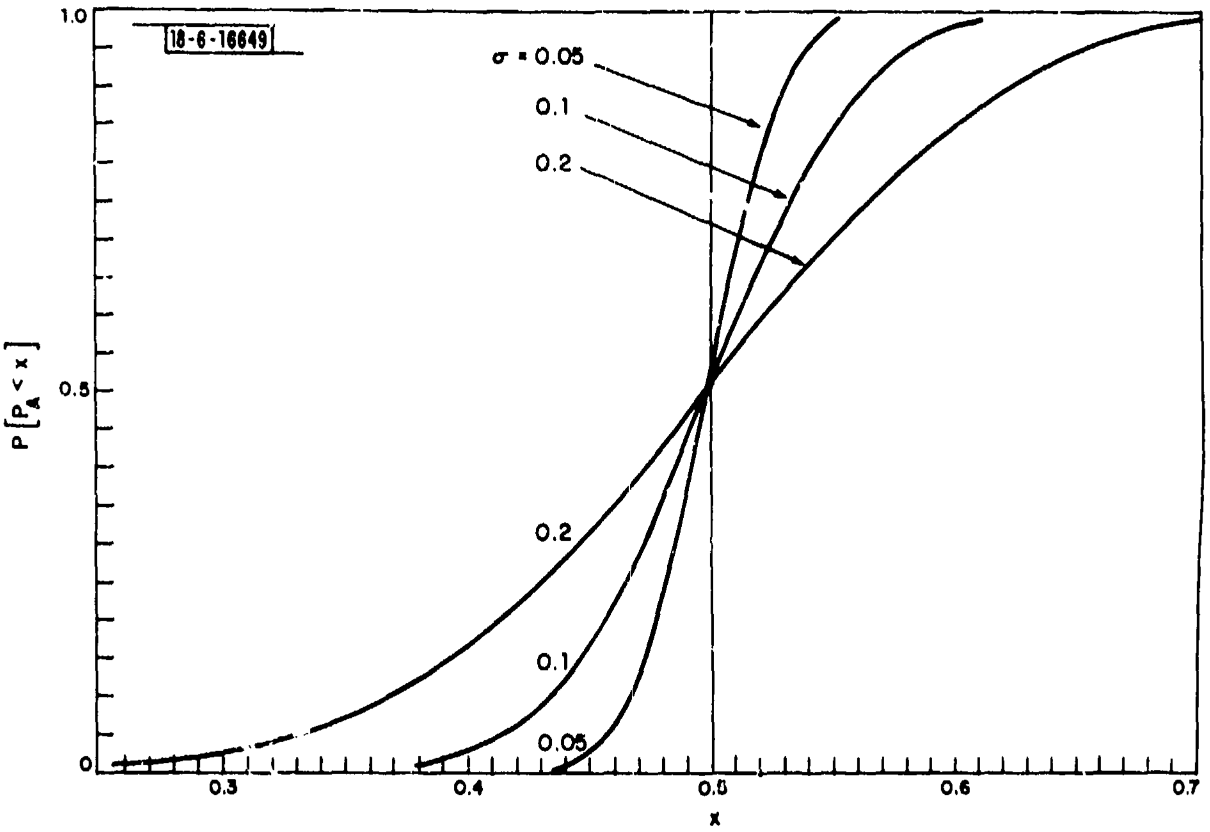

Fig. 5. Cumulative distribution of P_A for $P_A' = 0.5$

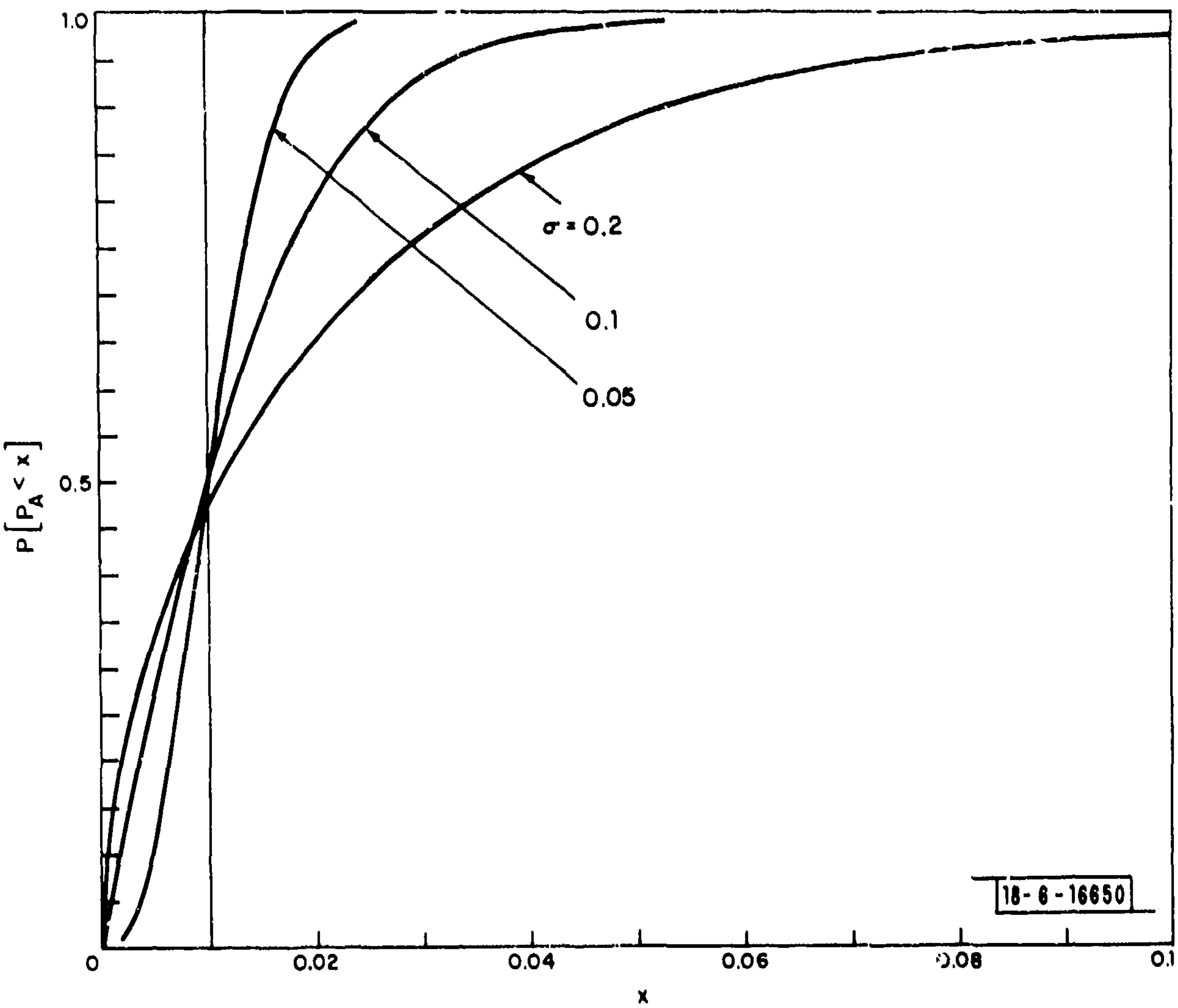

Fig. 6. Cumulative distribution of P_A for $P_A' = 0.99$

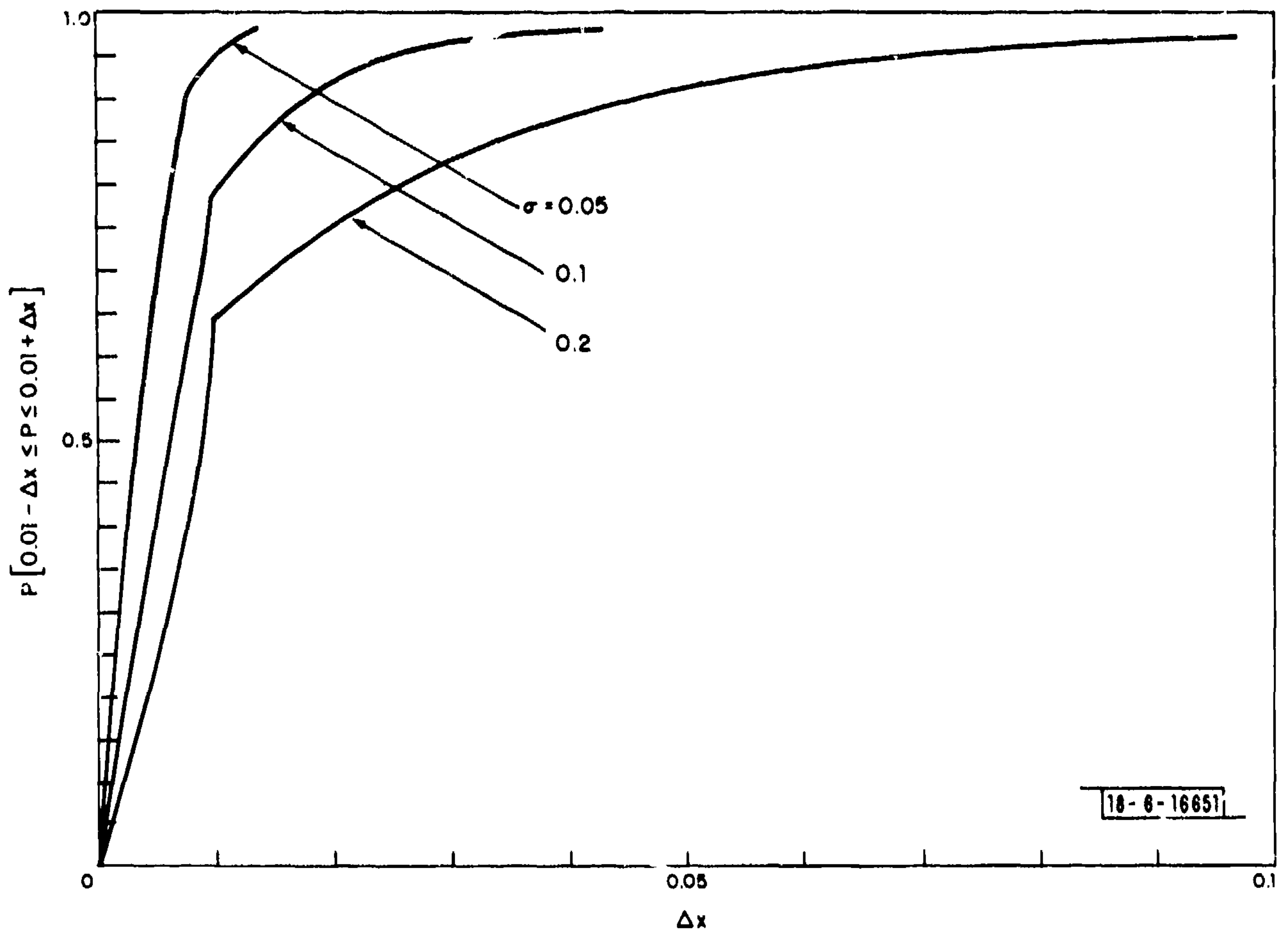

Fig. 7. Probability of P_A for a given error in P_A' (P_A' = 0.01).

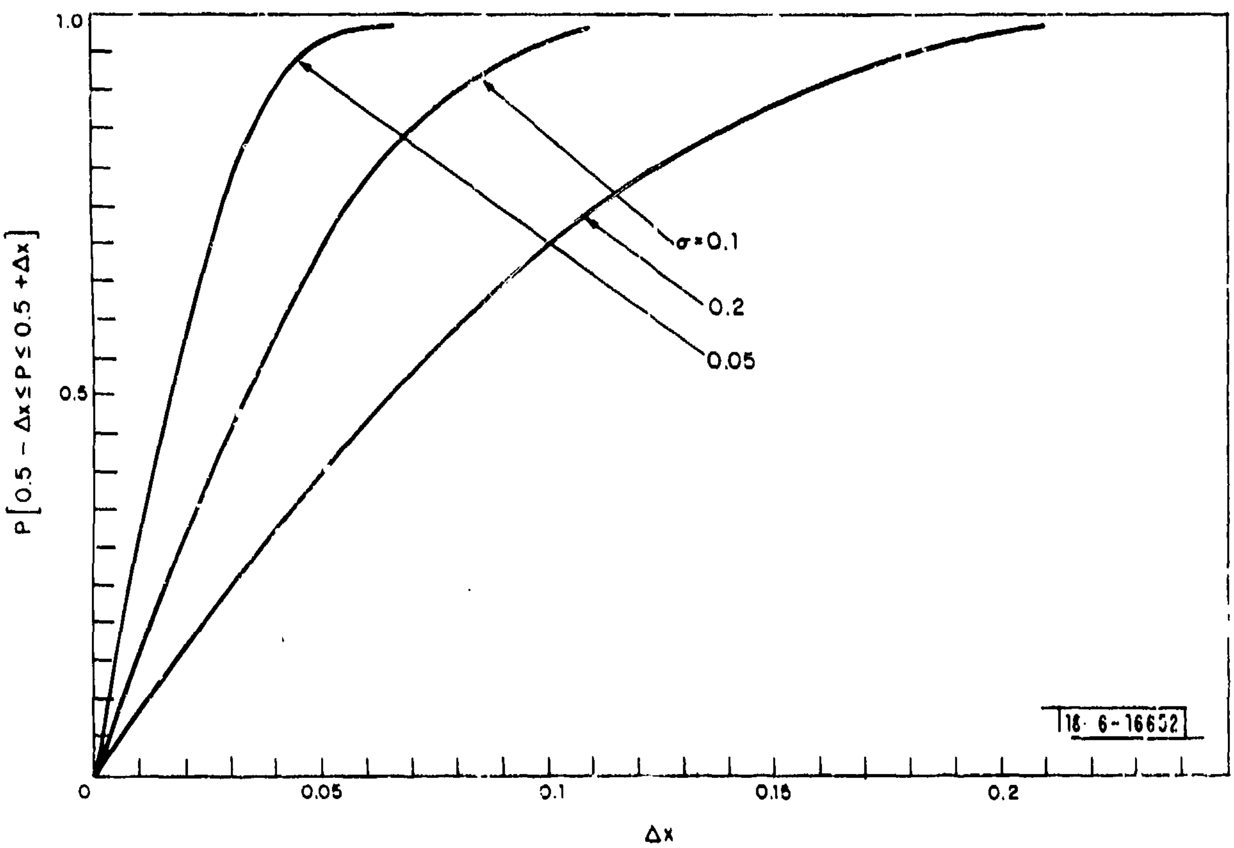

Fig. 8. Probability of P_A for a given error in P_A' ($P_A' = 0.5$).

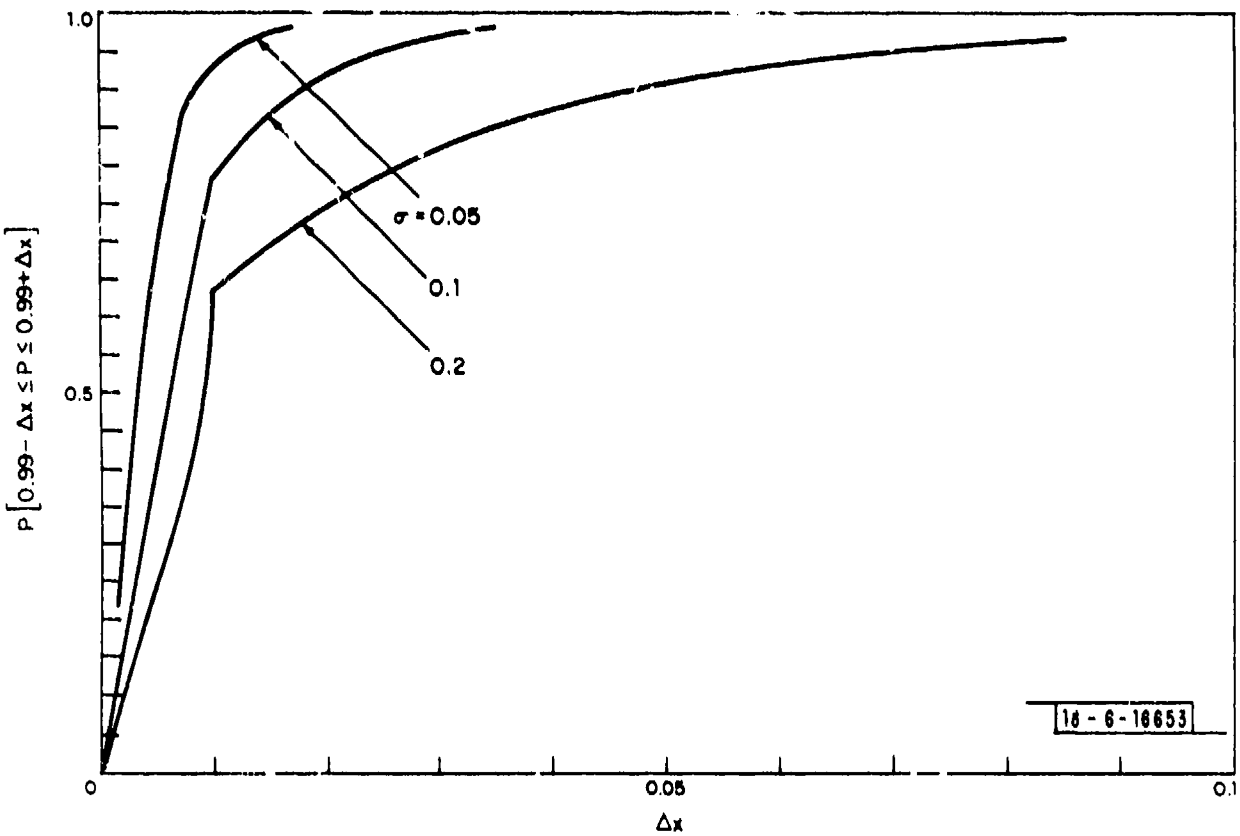

Fig. 9. Probability of P_A for a given error in P_A' ($P_A' = 0.99$).

the mean value of P_A, $\overline{P}_A$, is ≈ 0.011!

The knee in the curves shown in Fig. 7 and Fig. 9 is due to the asymmetry in the cumulative distribution. In particular the difference in slope for $x > P_A'$ to that for x less than P_A' causes the abrupt change in the slope of the curves (Figs. 7 and 8) since they are computed from the cumulative distribution.

Uniformly Distributed Errors

Let us next assume that the errors in Eq. (5) or (5a) have a uniform distribution instead of the previously assumed Gaussian distribution. It can be shown that, if $\phi(\tau_\ell)$ is distributed uniformly within $\overline{\phi}(\tau_\ell) \pm \phi_o$, $\xi = \phi(\tau_\ell) - \phi(\tau_m)$ will have a mean value $\overline{\xi} = \overline{\phi}(\tau_\ell) - \overline{\phi}(\tau_m)$ and it will have a triangular distribution between $\overline{\xi} \pm 2\phi_o$. In particular, the pdf becomes

$$p_\xi = \left(1 + \frac{u-\overline{\xi}}{2\phi_o}\right)/2\phi_o; \qquad -2\phi_o < u - \overline{\xi} < 0$$

$$= \left(1 - \frac{u-\overline{\xi}}{2\phi_o}\right)/2\phi_o; \qquad 0 < u - \overline{\xi} < 2\phi_o \tag{28}$$

Using (13) and (14)

$$P[P_A < x] = \frac{1}{4\phi_o^2}\left[\int_{\xi_x'}^{\xi_x} (2\phi_o - \overline{\xi} + u)\,du + \varepsilon_1 \int_0^{\xi_x} (2\phi_o - \overline{\xi} - u)\,du\right] \tag{29}$$

when $\qquad \xi_x < \overline{\xi}$ (29a)

$\qquad \xi_x' = \overline{\xi} - 2\phi_o$, $\varepsilon_1 = 0$ if $\overline{\xi} > 2\phi_o$. (29b)

If $\overline{\xi} < 2\phi_o$,

$$\xi_x' = 0 \tag{29c}$$

$$\varepsilon_1 = 1 \text{ if } \xi_x < 2\phi_o - \overline{\xi} \tag{29d}$$

$$\varepsilon_1 = 0 \text{ if } \xi_x > 2\phi_o - \overline{\xi} \quad . \tag{29e}$$

when $\xi_x \geq \overline{\xi}$

$$P[P_A < x] = \frac{1}{4\phi_o^2} \int_{\overline{\xi}}^{\xi_x} (2\phi_o - \overline{\xi} - u)du + P[P_A < \sin^2 \frac{\overline{\xi}}{2}], \tag{30}$$

where $\overline{\xi} \leq \pi - 2\phi_o$ which corresponds to $x < \cos^2\phi_o$. It is also necessary that $\xi_x \leq \overline{\xi} + 2\phi_o$. In order to calculate $P[P_A < x]$ when $\pi - 2\phi_o < \overline{\xi} < \pi$, we can use (29) for $\xi_x < \overline{\xi}$ and an appropriate modification of (30). Because this is not a particularly interesting case (i.e., $\overline{\xi} > \pi - 2\phi_o$) it will not be treated here.

Integrating (29) and (30) requires attention to the conditions indicated. Let us first consider those cases where $2\phi_o < \overline{\xi} < \pi - 2\phi_o$. For $\xi_x < \overline{\xi}$, let $\xi'_x = \overline{\xi} - 2\phi_o$; then

$$P[P_A < x] = \frac{1}{8\phi_o^2} (\xi_x - \xi'_x)^2. \tag{31}$$

For $\xi_x > \overline{\xi}$

$$P[P_A < x] = \frac{1}{2} + \frac{1}{8\phi_o^2} (4\phi_o - \xi_x + \overline{\xi}) (\xi_x - \overline{\xi}). \tag{31a}$$

When $\overline{\xi} < 2\phi_o$, we have

$$P[P_A < x] = \frac{\xi'_x \; x}{2\phi_o^2} \tag{32a}$$

for $0 < \xi_x < \overline{\xi}$ and $\xi_x < 2\phi_o - \overline{\xi}$. There are two possible conditions to consider; i.e., $\overline{\xi} \leq 2\phi_o - \overline{\xi}$ and $\overline{\xi} > 2\phi_o - \overline{\xi}$. Taking the latter case first

$$P[P_A < x] = \frac{\xi_x'^2}{2\phi_o^2} + \frac{1}{8\phi_o^2}\left[2\xi'(\xi_x - \xi'_x) + \xi_x^2 - \xi_x'^2\right] \qquad (32b)$$

for $2\phi_o - \xi < \xi_x < \overline{\xi}$ and

$$P[P_A < x] = \frac{1}{2} + \frac{1}{8\phi_o^2}\left[2\xi'_x(\xi_x - \xi'_x) + \xi_x^2 - \overline{\xi}^2\right] \qquad (32c)$$

for $\overline{\xi} < \xi \leq \xi''_x$, where $\xi''_x = 2\phi_o + \overline{\xi}$.

Now considering the case $\overline{\xi} \leq \xi'_x$, we have

$$P[P_A < x] = \frac{1}{4\phi_o^2}\left[4\phi_o\xi_x - \xi_x^2 + \overline{\xi}^2\right] \qquad (32d)$$

for $\overline{\xi} < \xi_x < \xi'$ and

$$P[P_A < x] = \frac{1}{8\phi_o^2}\left[\xi_x'^2 - 2\overline{\xi}^2 + 2\xi''_x\xi_x - \xi_x^2\right]. \qquad (32e)$$

when $\xi'_x < \xi_x < \xi''_x$.

A plot of $P[P_A < x]$ for $\overline{\xi} = \pi/2$ ($P'_A = 0.5$) and $\phi_o = 0.1$ radian is shown in Fig. 10. Notice the similarity between this curve and that for $\sigma = .1$ in Fig. 4. As should be expected, uniformly distributed errors with a maximum value $\phi_o = \sigma$, the RMS value of errors with a Gaussian distribution, are more likely to produce the desired value of P_A. This present analysis shows the possible effects should the actual errors be distributed either as

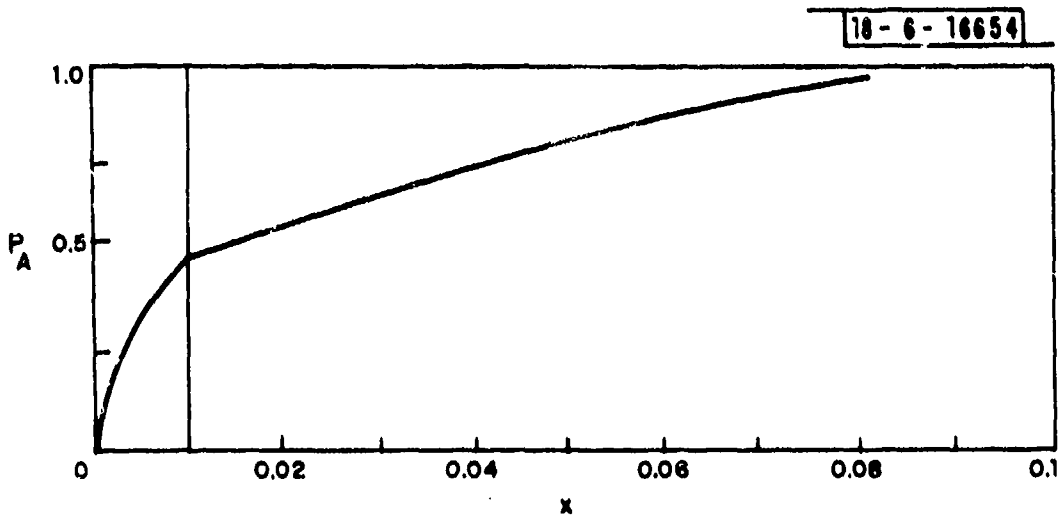

Fig. 10. Cumulative distribution of P_A for $P_A' = 0.5$.

Gaussian or a finite uniform distribution. Using the foregoing analysis, it is also interesting to note that for $\phi_o = 0.1$ radian and uniformly distributed errors, an attempt to set $P_A \leq 0.01$ will be successful only 50% of the time.

Conclusions

Should the measured data show that the errors have a Gaussian distribution and they have an RMS error = 5° ($\approx$ .1 radian) one can expect, from Fig. 8 ($P'_A = .99$), that 80% of the time the power delivered to the desired port (say port A) will be within 0.05 dB of the maximum power available. At the same time the power delivered to P_B will be more than 17 dB below power into the variable power divider. Similarly from Fig. 7, for equal power division ($P'_A = 0.5$), 80% of the time the P_A will be within 0.3 dB of the desired value. From these examples we see that only when trying to minimize P_A (or P_B) does the variation of $\phi(\tau)$ effect the power division significantly. Use of Eq. (32a), with $\bar{\xi} = 0$ ($P'_A = 0$), indicates that, with the assumed 5° RMS error in ϕ, P_A will be less than -30 dB (referred to the input power) with a probability of 0.62. This operational performance may have questionable value in a nulling antenna when null depth greater than 20 dB is desired.

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